This test is in two parts. On part one, you may not use a calculator; on part two, a (non-graphing) calculator is necessary. When you complete part one, you turn it in and get part two. Once you have turned in part one, you may not go back to it. You will show all work on the test paper, no scratch paper is allowed.

PART ONE - NO CALCULATORS ALLOWED

(1) Find each of the following: (Note: here, answers to inverse trig. problems should be in radians, not degrees)

(2 points each)

(a) $\tan^{-1}\left(\sqrt{3}\right) = \dfrac{\pi}{3}$

(b) $\sin\left(\dfrac{5\pi}{6}\right) \dfrac{1}{2}$

(c) $\cos^{-1}\left(-\dfrac{\sqrt{2}}{2}\right) = \dfrac{3\pi}{4}$

(d) $\sin^{-1}\left(-\dfrac{1}{2}\right) = \dfrac{-\pi}{6}$

(e) $\sin(120°) = \dfrac{\sqrt{3}}{2}$

(f) $\cos(150°) = \dfrac{-\sqrt{3}}{2}$

(g) $\cos^{-1}(0) \dfrac{\pi}{2}$

(h) $\tan^{-1}(-1) = \dfrac{-\pi}{4}$

(i) $\tan\left(\dfrac{\pi}{2}\right) = $ **undefined**

(j) $\sin^{-1}(0) = 0$

(k) $\cos^{-1}(5) = $ **undefined**

(l) $\tan(135°) = -1$

(2) HOW MANY solutions does each of the following equations with the given restrictions on θ have? (Do not need to solve, just tell how many solutions there would be.)

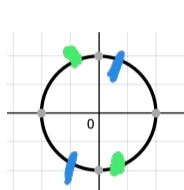(1 point each)

(a) $\sin\theta = 0.3;\quad -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$ **1**

(c) $\theta = \cos^{-1}\left(\dfrac{1}{3}\right)$ **1**

(b) $\tan\theta = \dfrac{1}{8}$ **Inf. many**

(d) $\tan\theta = \dfrac{5}{7};\ 0 \le \theta < 2\pi$ **2**

(3) Solve the following equations exactly. (all solutions)   (3 points each)
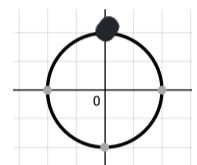
(a) $\tan^2\theta - 3 = 0$

(b) $\sin(4x) = 1$ **Cannot divide out this 4.**

$\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$

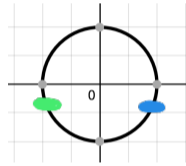$\theta = \dfrac{\pi}{3} + \pi k, \ \dfrac{2\pi}{3} + \pi k$

K integer

$4x = \dfrac{\pi}{2} + 2\pi k$

$x = \dfrac{\pi}{8} + \dfrac{\pi k}{2}$

K integer

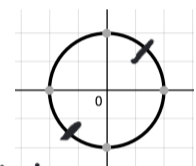(4) Solve the following equations exactly for $0 \le \theta \le 2\pi$.   (3 points each)

(a) $6\sin\theta + 3 = 0$

(b) $\sin(3x) = \dfrac{\sqrt{3}}{2}$ **period is small, go around 3 times "**

$6\sin\theta = -3$

$\sin\theta = -\dfrac{1}{2}$
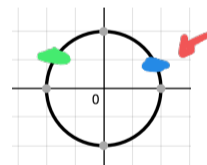
$\theta = \dfrac{7\pi}{6}, \ \dfrac{11\pi}{6}$

$\tan(3x) = 1$

$3x = \dfrac{\pi}{4} + \pi k$

$x = \dfrac{\pi}{12} + \dfrac{\pi k}{3} = \dfrac{\pi}{12} + \dfrac{4\pi k}{12}$

In $[0, 2\pi)$

$x = \dfrac{\pi}{12}, \ \dfrac{5\pi}{12}, \ \dfrac{9\pi}{12}, \ \dfrac{13\pi}{12}, \ \dfrac{17\pi}{12}, \ \dfrac{21\pi}{12}$
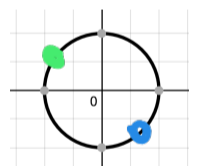
$\dfrac{3\pi}{4}$

$\dfrac{7\pi}{4}$

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(5) Solve the following equations exactly for $0 \le \theta \le 2\pi$.   (3 points each)
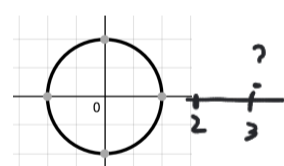
(a) $\sin\theta = \dfrac{1}{4}$

(b) $\tan\theta = -3$

(c) $\cos\theta = 3$

$\text{ref} = \sin^{-1}\dfrac{1}{4}$

$\theta = \sin^{-1}\dfrac{1}{4}, \ \pi - \sin^{-1}\dfrac{1}{4}$

$\text{ref} = \tan^{-1}(3)$

$\theta = \pi - \tan^{-1}(3)$

$2\pi - \tan^{-1}(3)$

no solution

$-1 \le \cos\theta \le 1$

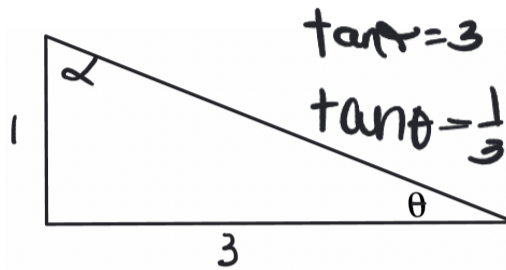* If you are going to use the reference angle approach, you need to use the **positive**

NAME: _____

MATH 8⎵⎵⎵⎵ Test 3
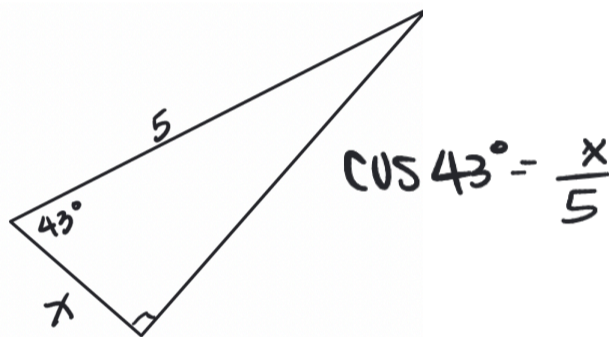PART TWO - CALCULATORS ALLOWED (no graphing calc.)
Show your work on this paper. EXACT answers are expected unless otherwise specified.

(1) Given the right triangle shown, solve for α and θ, exactly and give approximation in degrees (1 decimal point)
(6 points)

$\tan\alpha = 3$

$\tan\theta = \dfrac{1}{3}$

$\alpha = \tan^{-1}3 \approx 71.6°$

$\theta = \tan^{-1}\dfrac{1}{3} \approx 18.4°$

(2). Given the right triangle shown, solve for x exactly: *(2 points)*

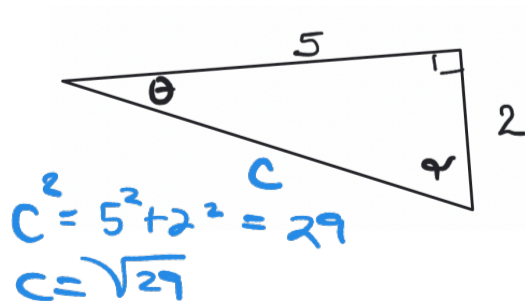$\cos 43° = \dfrac{x}{5}$

$x = 5\cos 43°$

(3). Find the value of the trig functions for the angle shown having the point (-3,1) on the terminal side (exact).

$r = \sqrt{(-3)^2 + 1^2} = \sqrt{10}$

(9 points)

2 pts each

$\sin(\theta) = \dfrac{1}{\sqrt{10}}$ $\csc(\theta) = \sqrt{10}$

$\cos(\theta) = \dfrac{-3}{\sqrt{10}}$ $\sec(\theta) = \dfrac{-\sqrt{10}}{3}$

$\tan(\theta) = \dfrac{-1}{3}$ $\cot(\theta) = -3$

1 pt each

$(-3, 1)$

(4). Find the value of the listed trig. function given the following right triangle. (notice different angles are being referenced) *(12 points)*
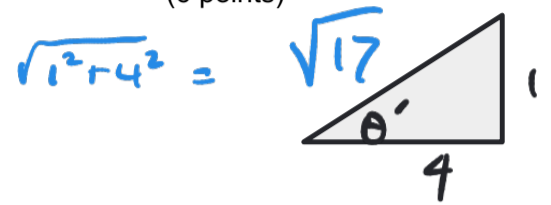
$c^2 = 5^2 + 2^2 = 29$

$c = \sqrt{29}$

$\sin(\theta) = \dfrac{2}{\sqrt{29}}$ $\cos(\alpha) = \dfrac{2}{\sqrt{29}}$

$\cos(\theta) = \dfrac{5}{\sqrt{29}}$ $\tan(\alpha) = \dfrac{5}{2}$

$\cot(\theta) = \dfrac{5}{2}$ $\csc(\alpha) = \dfrac{\sqrt{29}}{5}$

(5) Given that $\tan(\theta) = -\dfrac{1}{4}$ and θ is in Quadrant IV, find the values of the following exactly (show work)
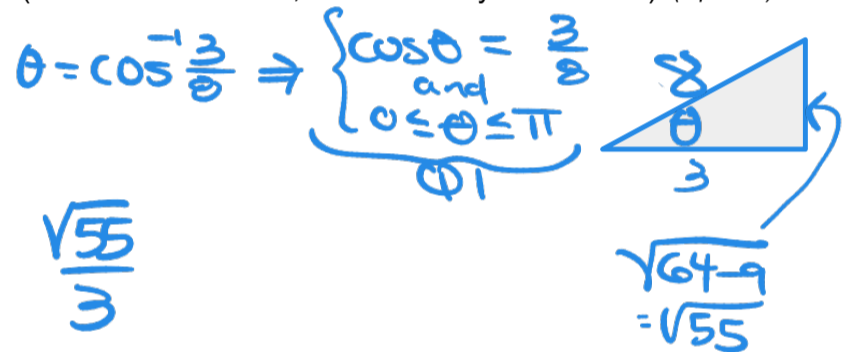
(6 points)

3 ways you can do this

$\sqrt{1^2+4^2} = \sqrt{17}$

$\tan\theta^c = \dfrac{1}{4} = \dfrac{opp}{adj}$

$\sin(\theta) = \underline{\dfrac{-1}{\sqrt{17}}}$ $\cos(\theta) = \underline{\dfrac{4}{\sqrt{17}}}$ $\cot(\theta) = \underline{-4}$

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(6) Evaluate exactly: $\tan\left(\cos^{-1}\left(\dfrac{3}{8}\right)\right)$ (You must show work, calculator may not be used). *(3 points)*

$\theta = \cos^{-1}\dfrac{3}{8} \Rightarrow \begin{cases} \cos\theta = \dfrac{3}{8} \\ and \\ 0 \le \theta \le \pi \end{cases}$

$\tan\left(\cos^{-1}\left(\dfrac{3}{8}\right)\right) = \tan\theta = \dfrac{\sqrt{55}}{3}$
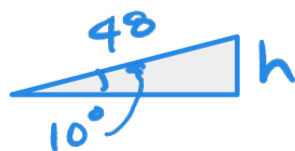
$\sqrt{64-9} = \sqrt{55}$

(7) A doctor is using a treadmill to assess the strength of a patient's heart. He sets the 48 inch long treadmill at an incline of $10°$, how high is the end of the treadmill raised? *(4points)*

$\dfrac{h}{48} = \sin 10°$
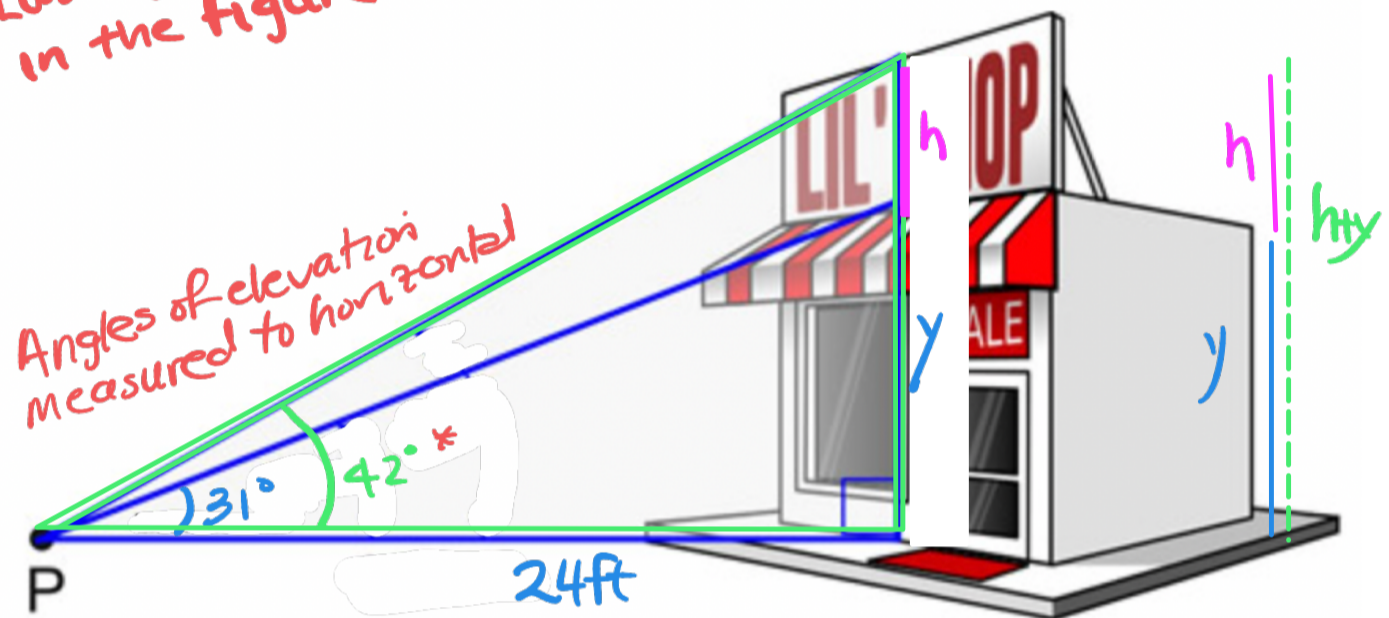
$h = 48 \sin 10°$

$h \approx 8.3$ inches

(8) Manny bought a new shop and wants to order a sign to replace the existing one. From a point P, he finds the angle of elevation of the roof, from ground level, to be $31°$ and the angle elevation of the top of the sign to be $42°$. If the Point P is 24 feet from the building, how tall is the sign. (solve exactly, then estimated to **three** decimal places)   *(9 points)*

Label your variables in the figure.

Angles of elevation measured to horizontal

$)31°$ $)42°$ *

P

24ft

$\tan 42° = \dfrac{h+y}{24}$

$24\tan 42° = h+y$

$h = 24\tan 42° - y$

$h = 24\tan 42° - 24\tan 31°$

$\tan 31° = \dfrac{y}{24}$

$y = 24\tan 31°$

$h = 24(\tan 42° - \tan 31°) \approx 7.189 \text{ inches}$